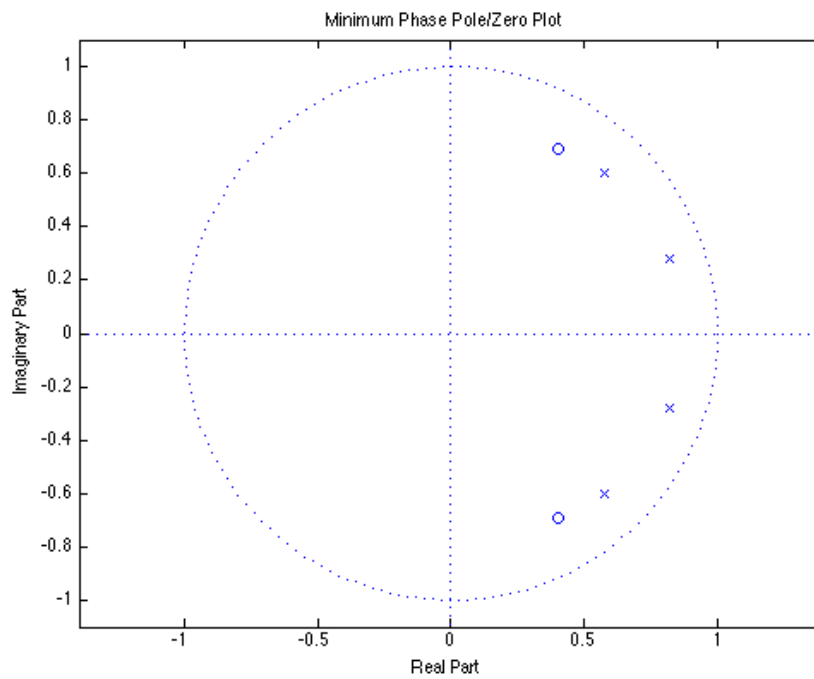
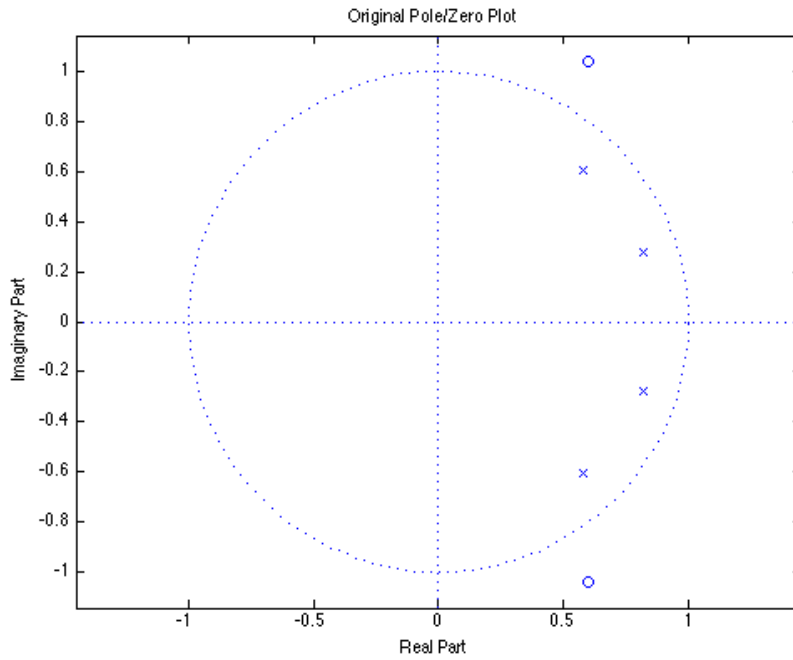


EEN 540 – Homework 2

Connor McCullough

Part A

Pole-Zero plots for original non-minimum phase filter (top) and minimum phase (bottom).



Part B

Non-Minimum Phase Transfer Function:

$a + jb$	$Re = r \cos \theta$	$Im = r \sin \theta$
$0.9e^{j\pi/8}$	$Re = .831$	$Im = .344$
$0.8e^{j3\pi/4}$	$Re = -.566$	$Im = .566$
$1.2e^{j\pi/3}$	$Re = .6$	$Im = 1.04$

$H(z) = \frac{(z - .6 + j1.04)(z - .6 - j1.04)}{(z - .566 + j.566)(z - .566 - j.566)(z - .831 + j.344)(z - .831 - j.344)}$
 $H(z) = \frac{z^2 - 1.2z + 1.44}{z^2 - 1.13z + .64} \cdot \frac{(z^2 - 1.66z + 0.81)^2}{z^2 - 1.6z + 0.52}$
 $\frac{z^4 - 1.66z^3 + .81z^2 - 1.13z^2 + 1.88z - 0.91z + .64z^2 - 1.06z + 0.52}{z^2 - 1.6z + 1.44}$
 $\frac{z^4 - 2.79z^3 + 3.33z^2 - 2z + 0.52}{z^2 - 1.6z + 1.44}$

Minimum Phase Transfer Function:

$a + jb$	$Re = r \cos \theta$	$Im = r \sin \theta$
$0.8e^{j\pi/3}$	$Re = .4$	$Im = .69$

$(z - .4 + j.69)(z - .4 - j.69)$
 $(z^2 - .8z + 1.6)$
 $\frac{z^2 - .8z + 1.6361}{z^4 - 2.79z^3 + 3.33z^2 - 2z + 0.52}$

Part C

Non-minimum Phase Fourier Transform:

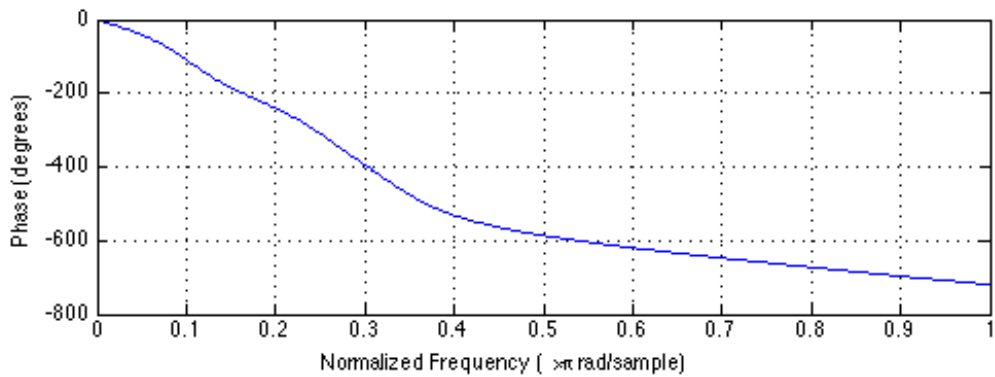
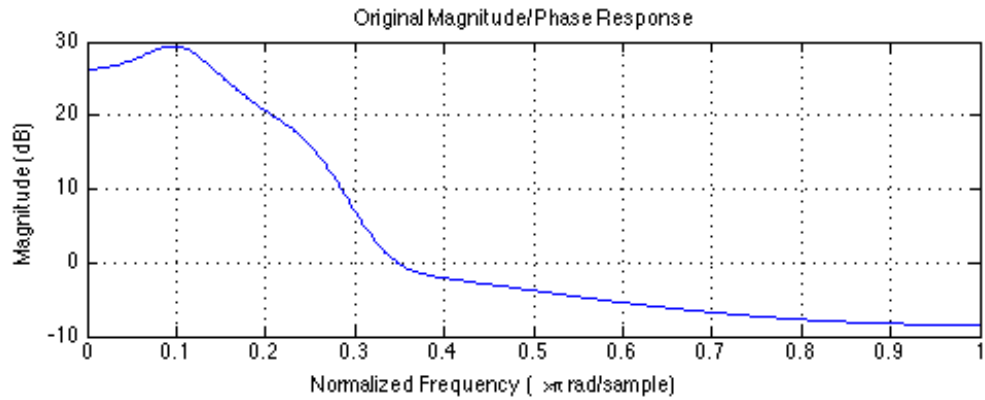
$H(\omega) = \frac{(e^{j\omega} - 1.04e^{j\pi/8})(e^{j\omega} - 1.04e^{-j\pi/8})}{(e^{j\omega} - 0.9e^{j\pi/3})(e^{j\omega} - 0.9e^{-j\pi/3})(e^{j\omega} - 0.8e^{j3\pi/4})(e^{j\omega} - 0.8e^{-j3\pi/4})}$
 $= \frac{(1 - 1.04e^{j\omega}e^{j\pi/8})(1 - 1.04e^{-j\omega}e^{-j\pi/8})}{(1 - 0.9e^{j\omega}e^{j\pi/3})(1 - 0.9e^{-j\omega}e^{-j\pi/3})(1 - 0.8e^{j\omega}e^{j3\pi/4})(1 - 0.8e^{-j\omega}e^{-j3\pi/4})}$
 $H(\omega) = \frac{(1 - 1.04e^{-j(\omega - \pi/8)})}{(1 - 0.9e^{-j(\omega - \pi/3)})(1 - 0.8e^{-j(\omega - 3\pi/4)})}$

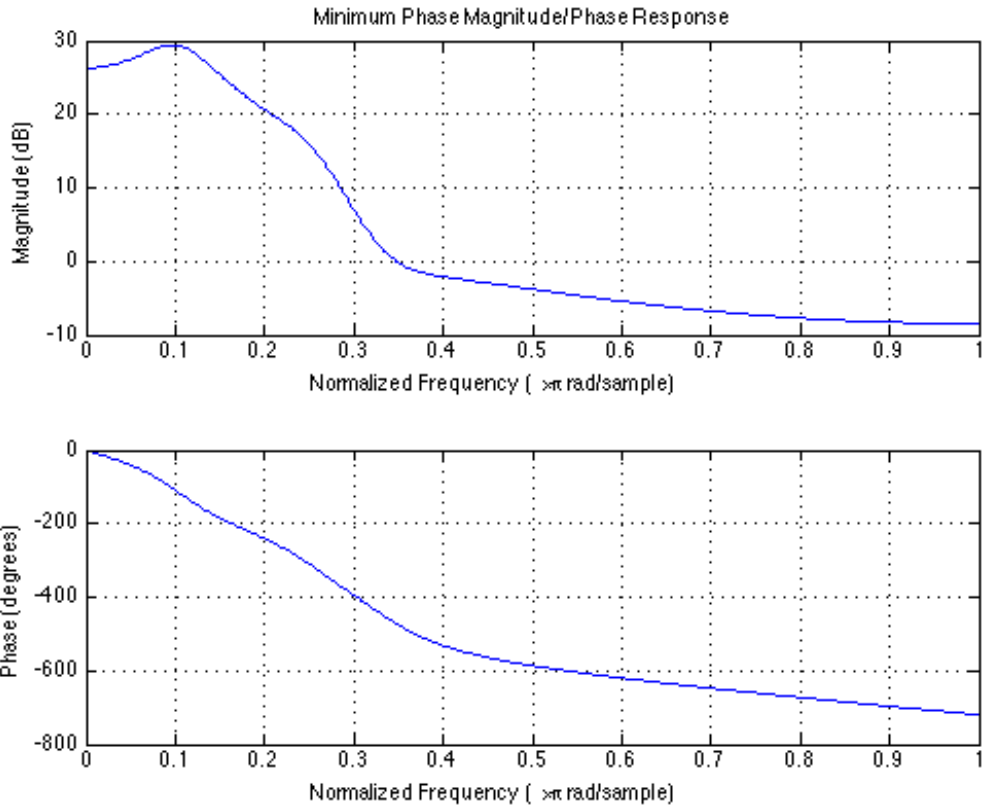
Minimum Phase Transfer Function:

$$C \quad H(\omega) = \frac{1 - 0.8e^{-j(\omega \pm \pi/3)}}{(1 - 0.9e^{-j(\omega \pm \pi/8)})(1 - 0.8e^{-j(\omega \pm \pi/4)})}$$

Part D

Magnitude and Phase Response of non-minimum phase filter (top) and minimum phase filter (bottom).





Part E

Non-minimum phase difference equation:

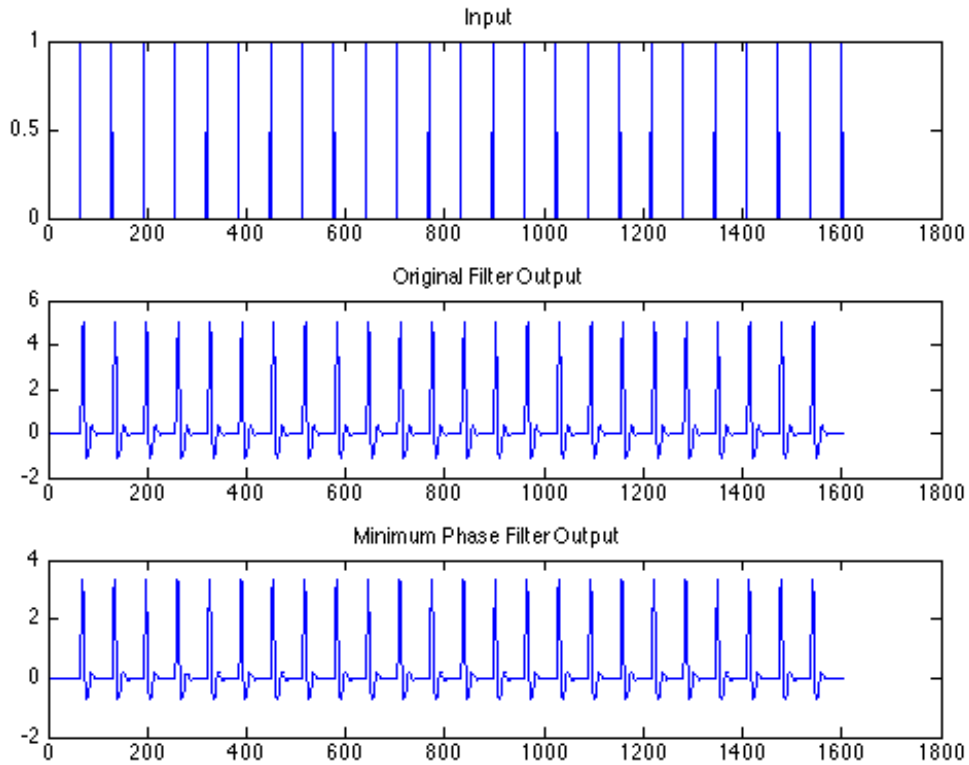
$$\begin{aligned}
 \text{e) } \frac{Y(z)}{X(z)} &= \frac{z^2 - 1.2z + 1.44}{z^4 - 2.79z^3 + 3.33z^2 - 2z + 0.52} \\
 Y(z)[z^4 - 2.79z^3 + 3.33z^2 - 2z + 0.52] &= X(z)[z^2 - 1.2z + 1.44] \\
 Y(z)[1 - 2.79z^{-1} + 3.33z^{-2} - 2z^{-3} + 0.52z^{-4}] &= X(z)[1 - 1.2z^{-1} + 1.44z^{-2}] \\
 y[n] - 2.79y[n-1] + 3.33y[n-2] - 2y[n-3] + 0.52y[n-4] &= x[n] - 1.2x[n-1] + 1.44x[n-2] \\
 y[n] &= 2.79y[n-1] - 3.33y[n-2] + 2y[n-3] - 0.52y[n-4] + x[n] - 1.2x[n-1] + 1.44x[n-2]
 \end{aligned}$$

Minimum phase difference equation:

$$\begin{aligned}
 \text{e)} \quad Y(z) &= \frac{z^2 - 0.8z + 0.6361}{X(z) = \frac{z^4 - 2.79z^3 + 3.33z^2 - 2z + 0.52}{Y(z)[z^2 - 0.8z + 0.6361] = X(z)[z^4 - 2.79z^3 + 3.33z^2 - 2z + 0.52]} \\
 Y(z)[1 - 0.8z^{-1} + 0.6361z^{-2}] &= X(z)[1 - 2.79z^{-1} + 3.33z^{-2} - 2z^{-3} + 0.52z^{-4}] \\
 y[n] - 0.8y[n-1] + 0.6361y[n-2] &= x[n] - 2.79x[n-1] + 3.33x[n-2] - 2x[n-3] + 0.52x[n-4] \\
 y[n] &= 0.8y[n-1] - 0.6361y[n-2] + x[n] - 2.79x[n-1] + 3.33x[n-2] - 2x[n-3] + 0.52x[n-4]
 \end{aligned}$$

Part F

Input/Output plot for non-minimum phase filter and minimum phase filter.



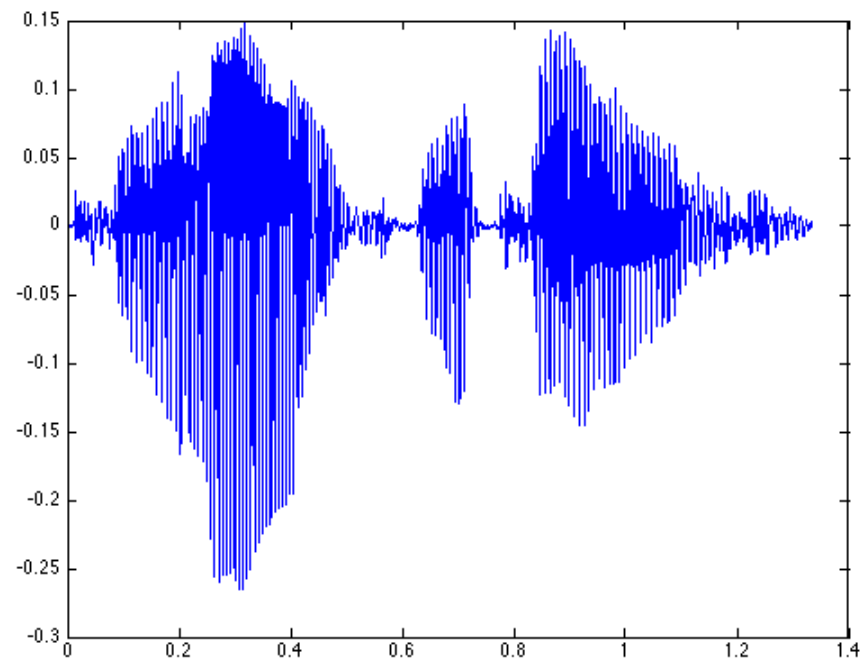
Problem 2

Connor McCullough:

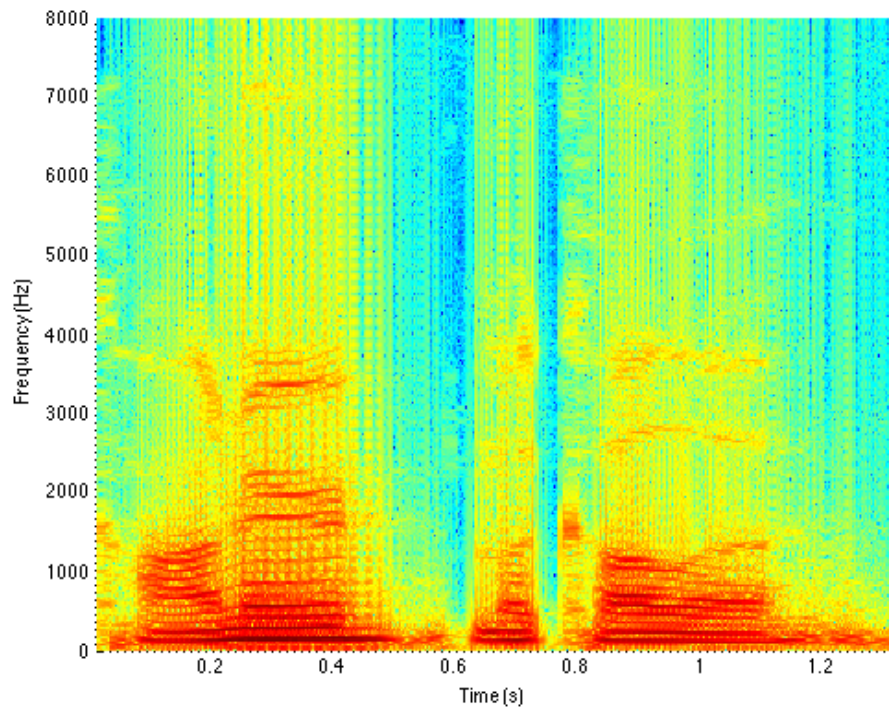
/k/ /a/ /n/ /R/

/m/ /l/ /k/ /A/ /l/ /A/

Time Domain Plot:



Narrow Band Spectrogram:



Wide Band Spectrogram:

