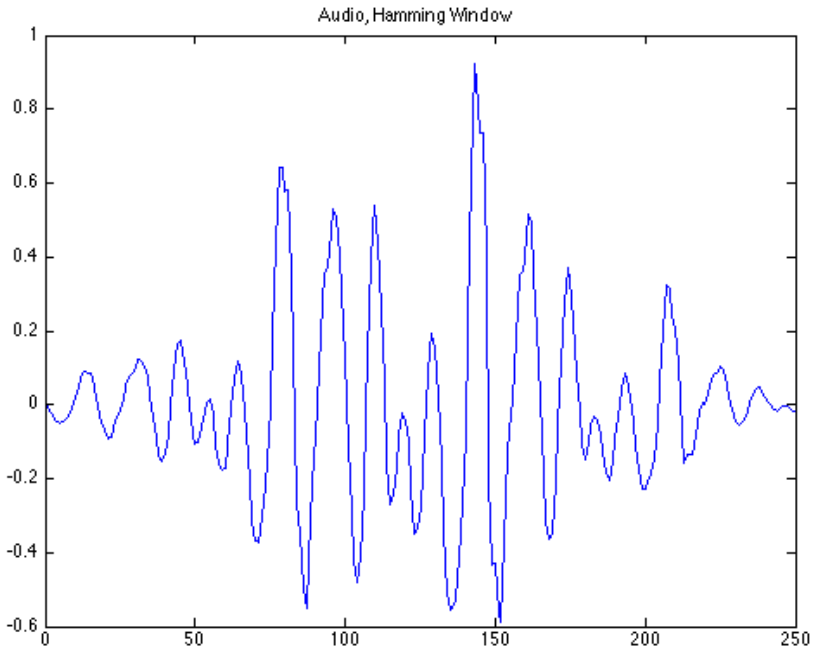
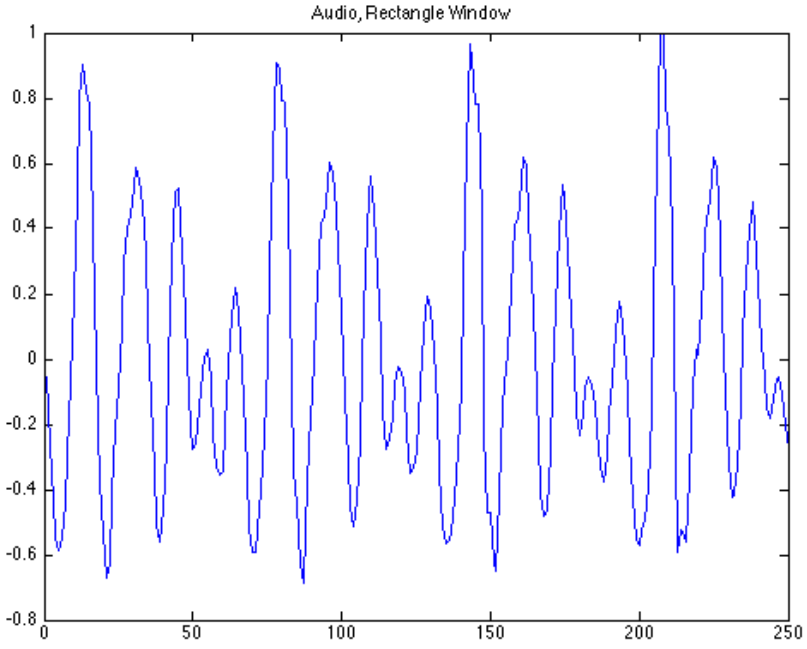
EEN 540 – Homework 1

Connor McCullough

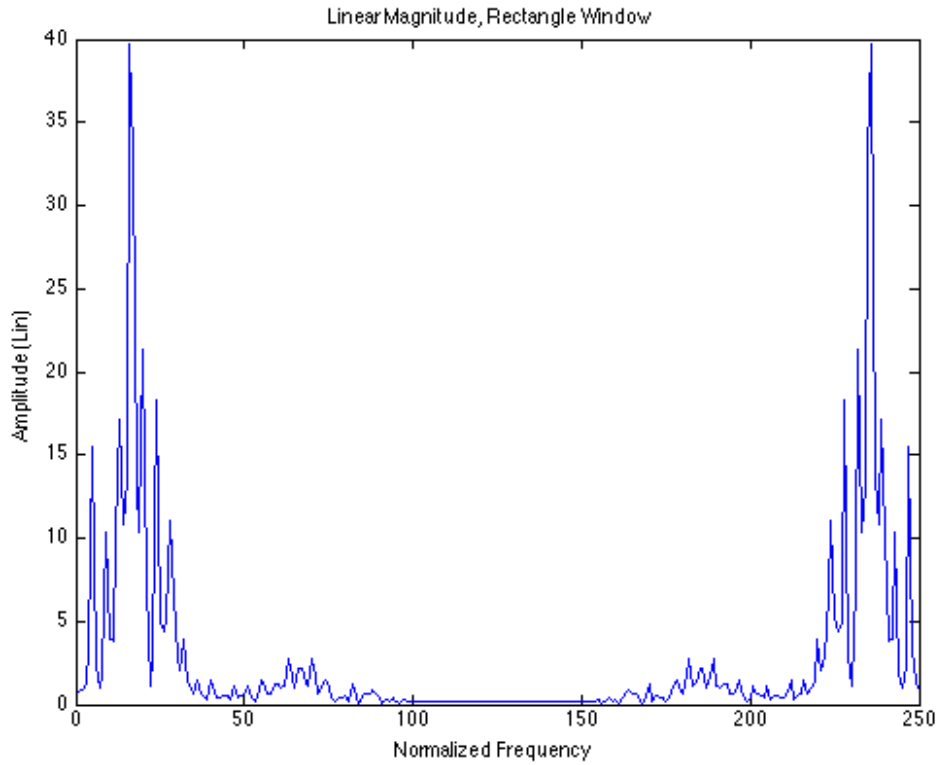
Part A

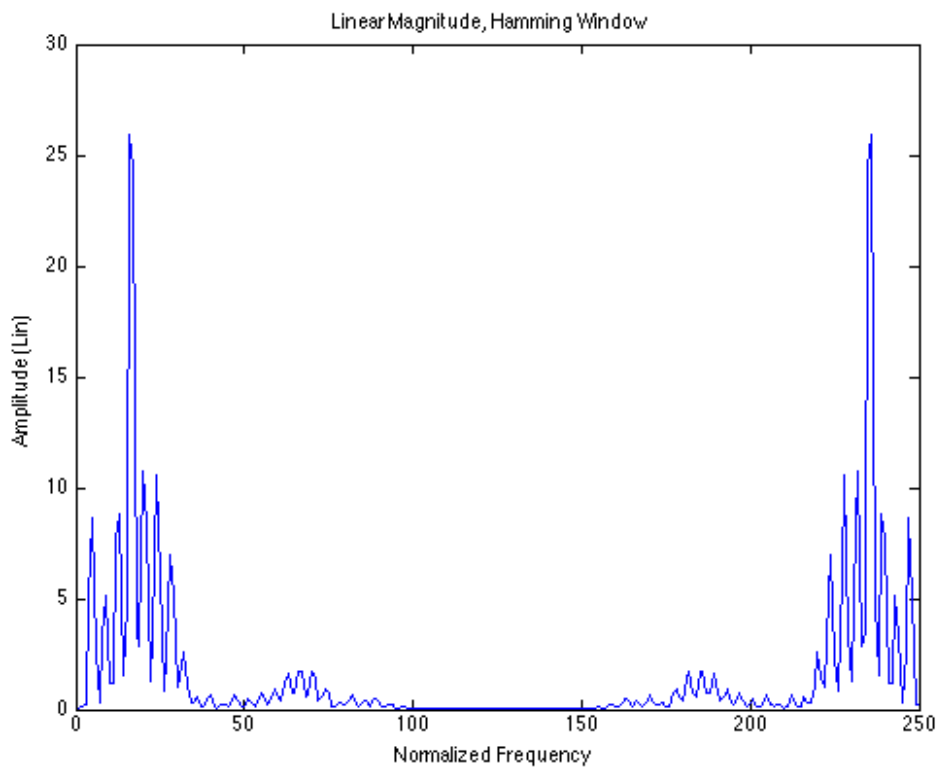
250 sample audio signal that is Hamming Windowed (top) and Rectangular Windowed (bottom)



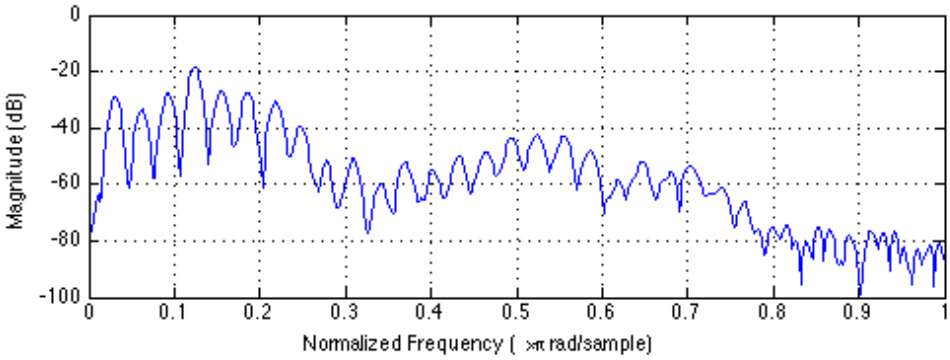


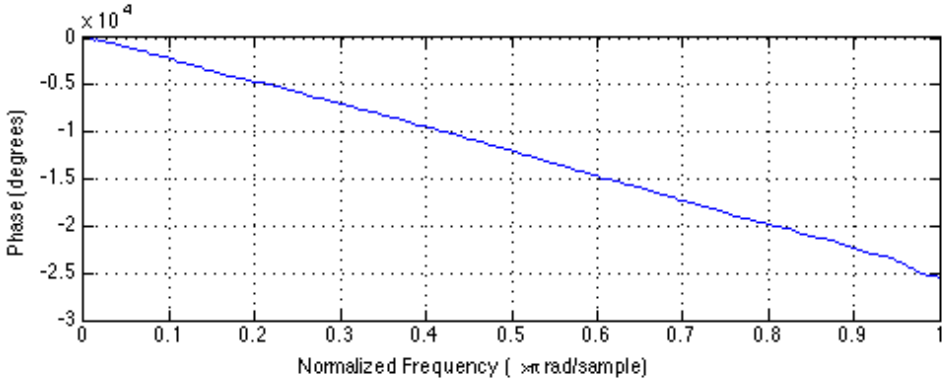
Magnitude (linear) of the Hamming Windowed signal and Rectangular Windowed signal:

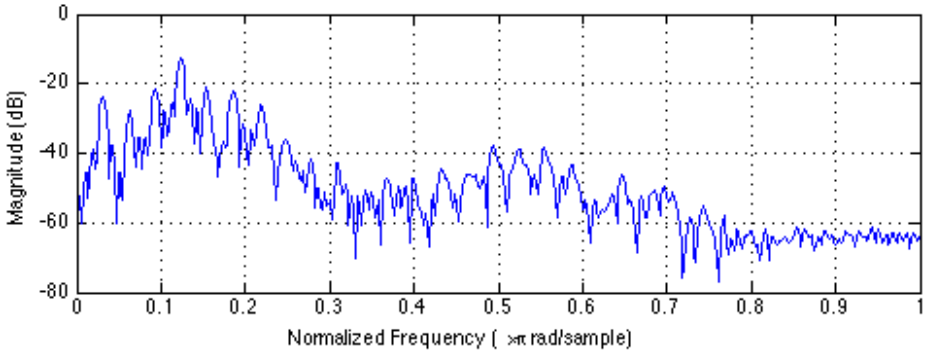


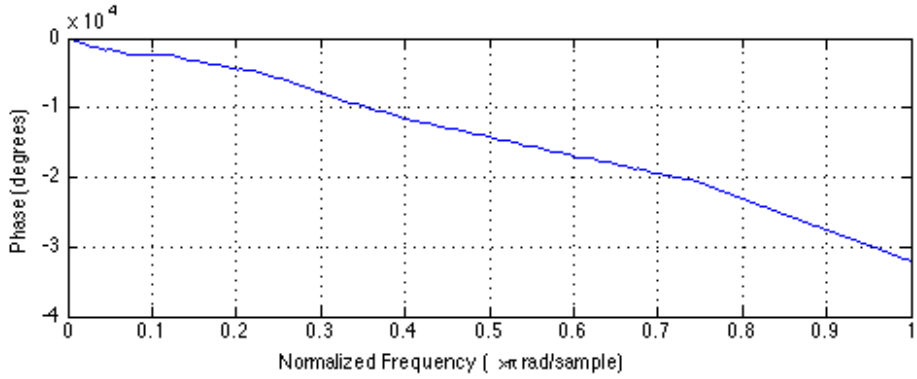


Magnitude (dB) and Phase of Hamming Windowed Signal and Rectangular Windowed signal:







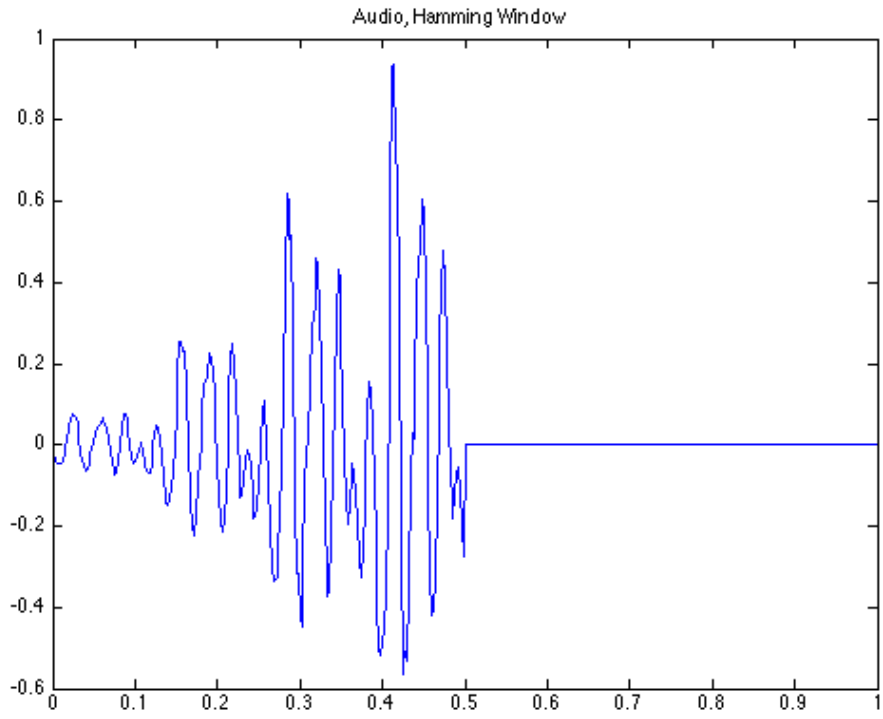


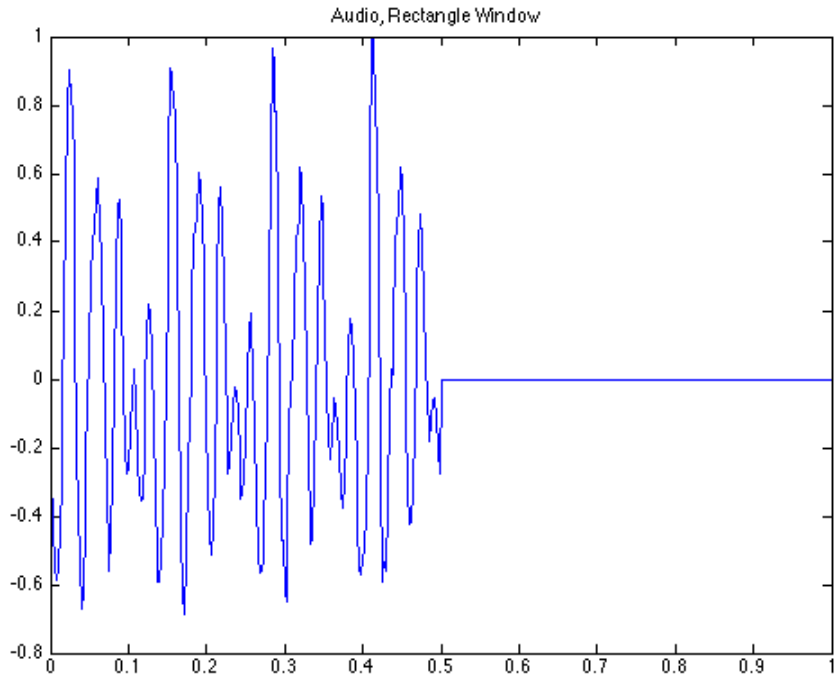
Analysis

The overall shape of the magnitude responses for both audio signals is the same. However, the Rectangular Windowed frequency response has much more noise than the Hamming Windowed frequency response. This is because the harsh discontinuities of the rectangular window add impulses to the signal. This adds more frequencies from every range, but in the same amount. The overall power of the FFT is higher for the Rectangular window, because the Hamming window attenuates the signal, removing some of its energy in the frequency domain. The phase response of the Hamming Windowed signal is much more close to linear phase than that of the Rectangular Windowed signal. This is due to the distortion added by the rectangular window. Since linear phase is more desirable, the Hamming Window is the preferred method for windowing signals.

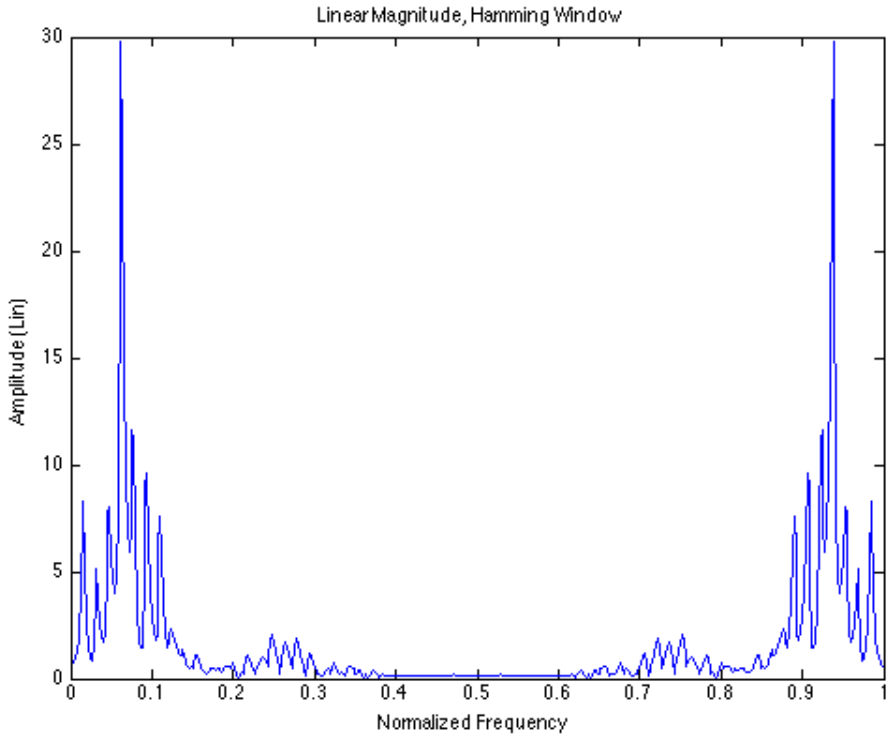
Part B

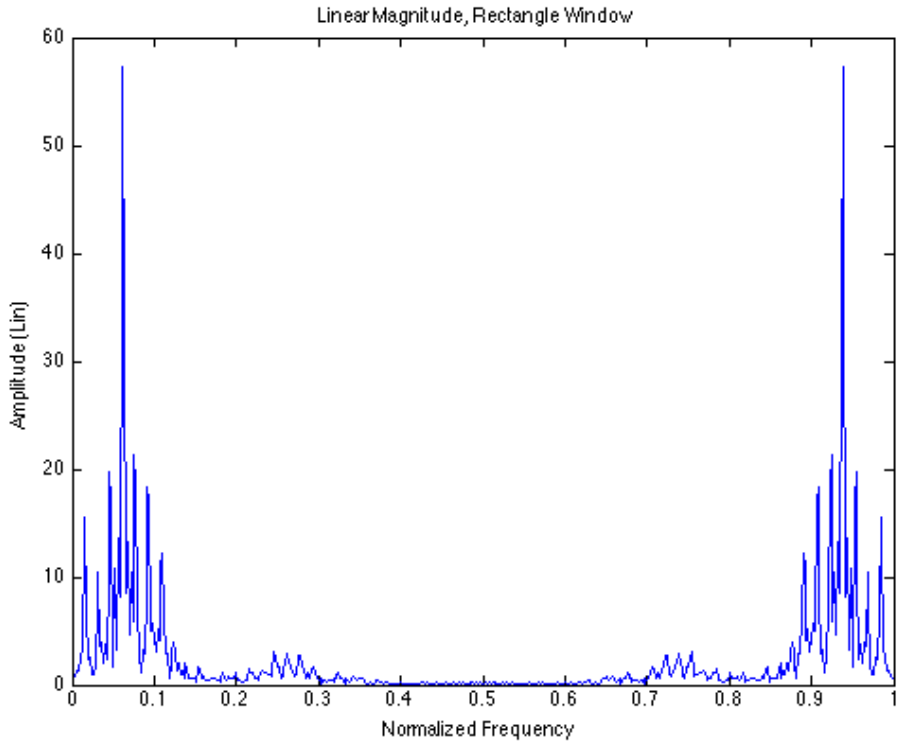
250 sample audio signal with 250 samples of zero padding at end



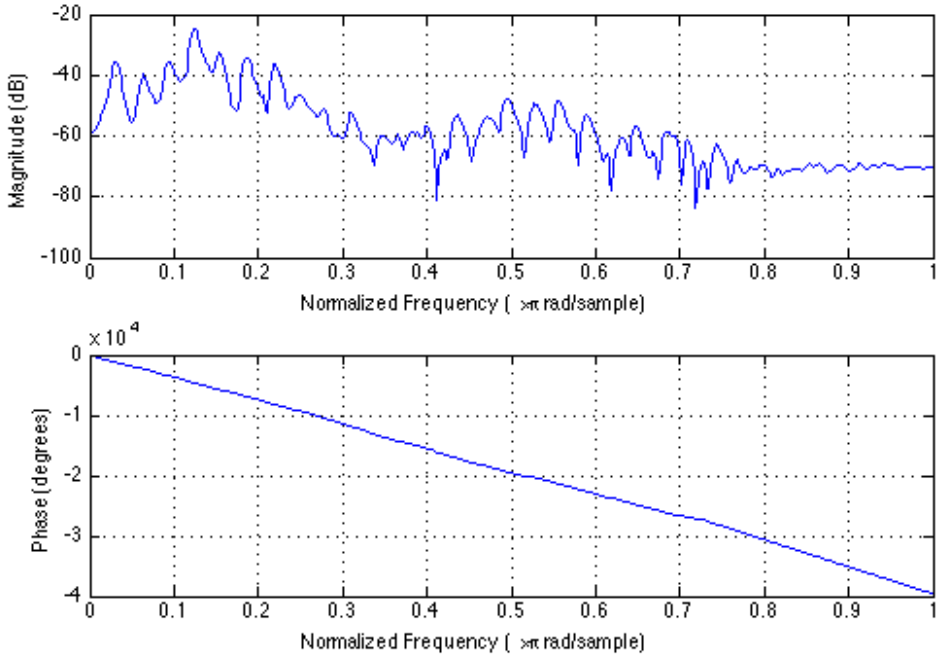


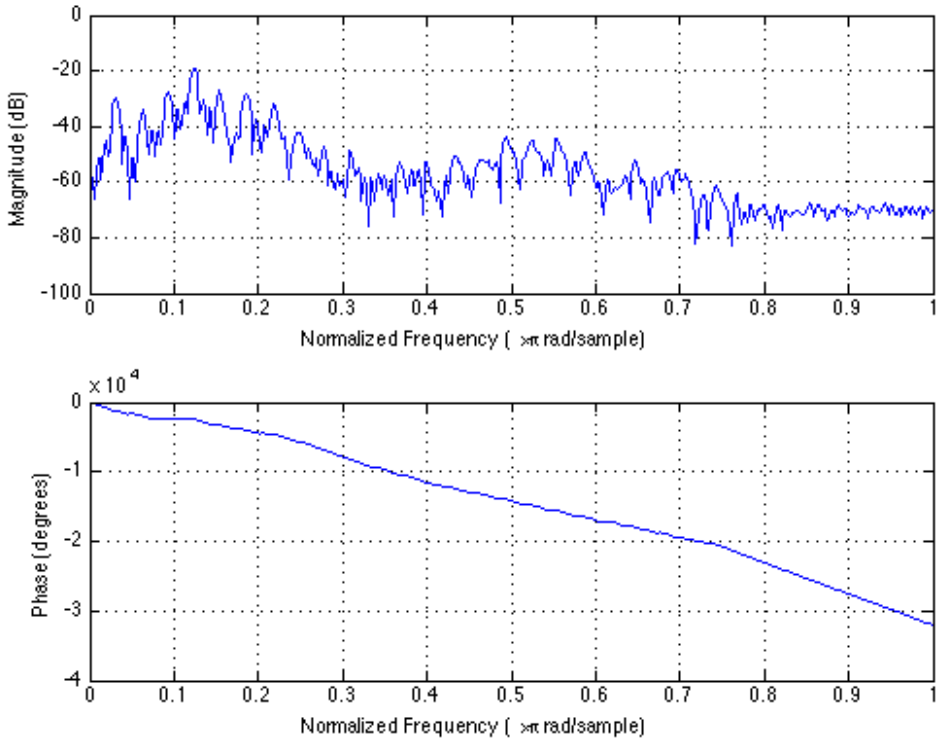
Magnitude (linear) of the Hamming Windowed signal and Rectangular Windowed signal:





Magnitude (dB) and Phase of Hamming Windowed Signal and Rectangular Windowed signal:





Analysis:

The results were similar with 250 samples of zero padding at the end, with a few differences. The main difference Is there appears to be less noise in the frequency response of the rectangular windowed signal, because there is no discontinuity at the end due to zero padding. The overall frequency response for the hamming and rectangular windowed signals has smaller ripples than in part A. The phase response is similar to the phase response from part A as well.

Problem 1

1) $W(\omega) = \sum_{n=-\infty}^{\infty} w[n] e^{-j\omega n}$

$w_R[n] = \begin{cases} 1, & 0 \le n \le N-1 \\ 0, & \text{elsewhere} \end{cases}$

$W(\omega) = \sum_{n=0}^{N-1} e^{-j\omega n}$, $\sum_{k=0}^{n} \gamma^k = \frac{1-\gamma^{n+1}}{1-\gamma}$, $\sum_{k=0}^{\infty} \gamma^k =$

$W(\omega) = \frac{1 - e^{-j\omega(N-1)}}{1 - e^{-j\omega}}$

a) $\lim_{\omega \to 0} W(\omega) = \frac{\cos(-\omega(N-1)) + 1}{\omega} = 2$

Problem 2:

2) $w_H[n] = \overbrace{(0.54 - 0.46\cos[2\pi n/(N-1)])}^{w_x[n]} \cdot w_R[n]$

$W_H[\omega] = F\{w_x[n] \cdot w_R[n]\}$

$= F\{w_x[n]\} * F\{w_R[n]\}$

$W_R[\omega] = \text{sinc}(N/2\omega - N/2)$

$W_x[\omega] = 0.54\delta - 0.46[\delta(\omega + 2\pi/(N-1) + 2\pi k) + \delta(\omega - 2\pi/(N-1) + 2\pi k)]$

$W_H[\omega] = \text{sinc}(N/2\omega - N/2) * [0.54\delta - 0.46[\delta(\omega + 2\pi/(N-1) + 2\pi k) + \delta(\omega - 2\pi/(N-1) + 2\pi k)]]$

$|W_H[0]| = \text{sinc}(-N/2) * 0.54 = 0.00918$

b) N samples

c) N/2 samples

$W_R(\omega)$ is a continuous function which converges to 0 as ω gets larger. $W_H(\omega)$ is a set of replicas of the sinc function centered at the fundamental frequency of the cosine signal, replicated every 2π radians.