

DFT Representation and Frequency Domain Filtering

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Abstract—Frequency Domain Filtering is a very important aspect of Image Processing, as it allows masking techniques such as blurring and sharpening, which edit the frequency content of an image. This project first looks at the individual effects that magnitude and phase have on the image. Next, different filters are applied to the image and their effects on the processed image are examined. Filter cutoffs are both chosen arbitrarily, as well as chosen based on maintaining a certain level of power in the entire image. The filters examined include ideal lowpass, bandpass, and high pass filters, and Butterworth low pass, high pass, and high boost.

Keywords—DFT, filtering, masking, ideal, Butterworth, low pass, high pass, band pass, high boost

I. INTRODUCTION (HEADING I)

Using a Fourier Transform on an image is a gateway to many image processing techniques involving convolution, as well as aiding in knowledge of signal processing concepts in general. Two filters commonly used to smooth and sharpen images are the ideal filter and Butterworth Filter. By analyzing these filters and their effects both in the frequency and x-y domain, the effects different filters can have across different domains can be easily seen.

II. EFFECTS OF MAGNITUDE AND PHASE ON IMAGE

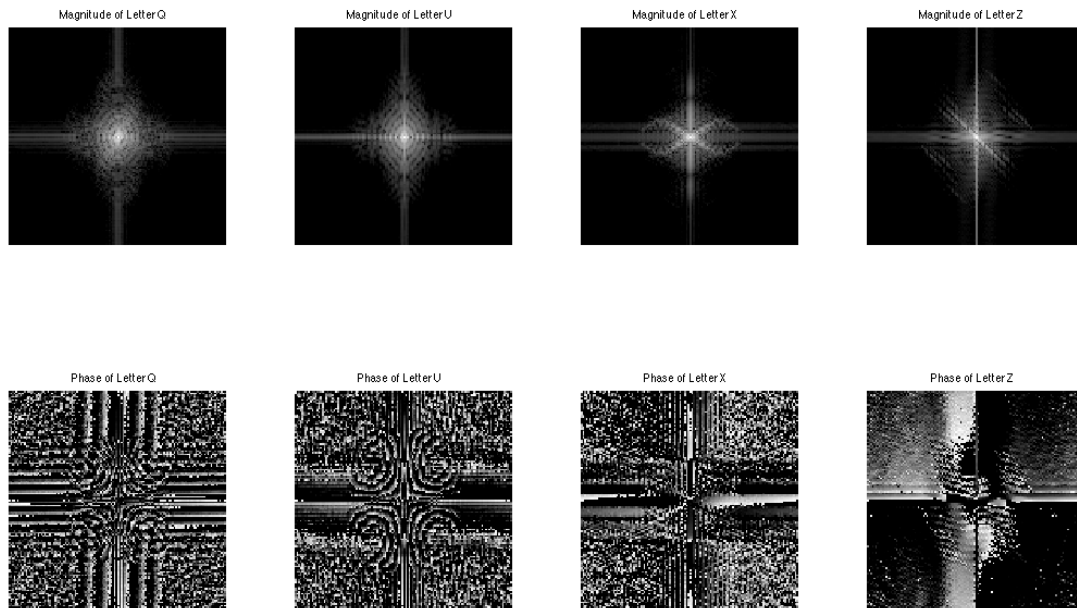


Fig. 1. Magnitude and Phase of Each letter from the given data set.

A. Keeping lowest frequency bin while other low bins

In this test, the center bin was maintained and a ring of frequency bins of $N 1:5$ around the center bin. The result is a mostly grey image with less contrast as more low frequency bins are zeroed out. The center bin can be seen as the “DC” bin in traditional signal processing and it creates an overall offset energy level. Instead of the frequency content being around 0, the center bin makes the average color grey, a middle level of intensity. While the outlines of the letter (high frequencies) are maintained, the overall contrast of the letter (low frequencies) is lowered. Because there is so much energy in the low bins, the effect is easily visible.

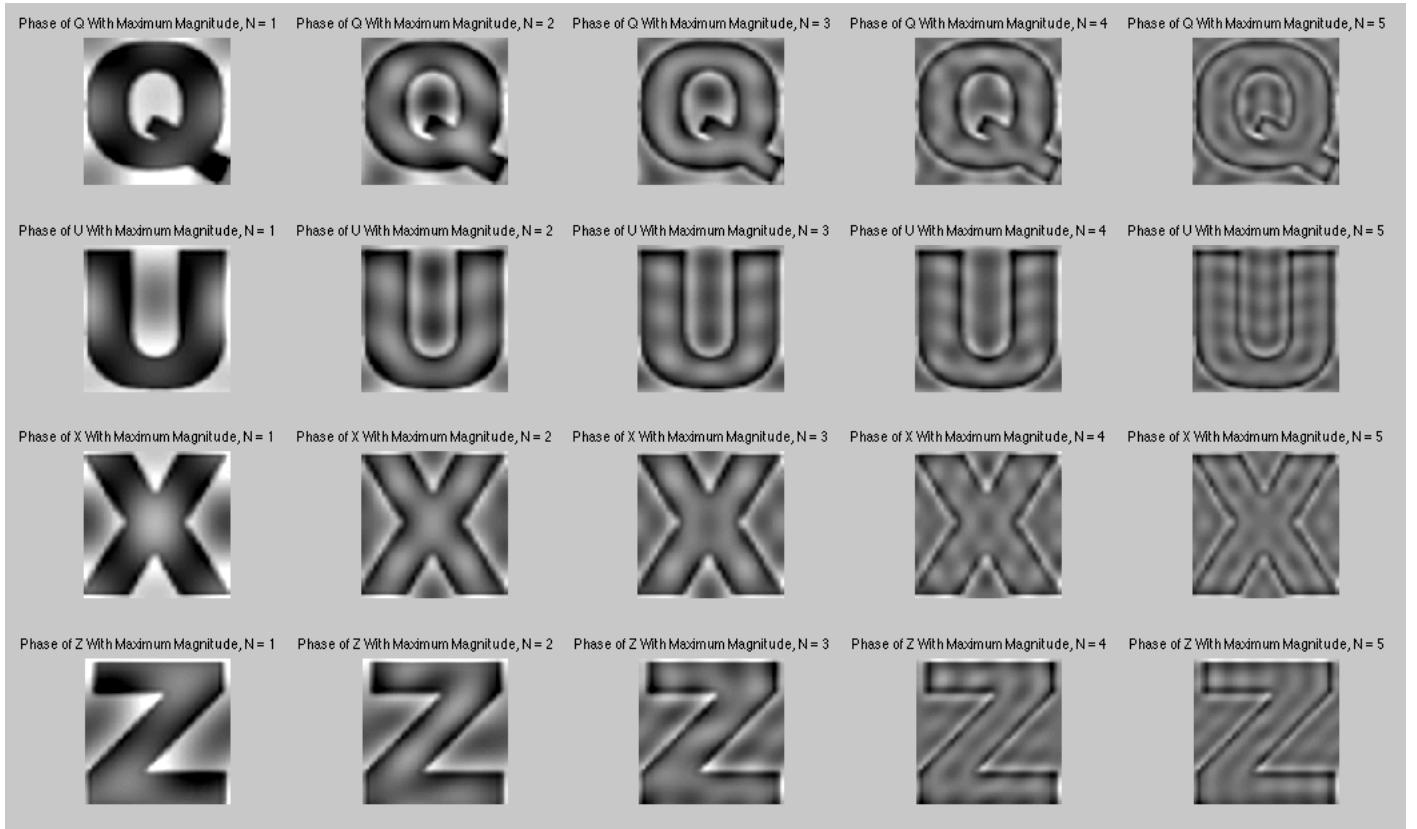


Fig. 2. Phase Maintained, center bin maintained, low frequencies N 1:5 removed.

B. Keep Magnitude, Zero Phase

When the phase is set to 0 but the magnitude is maintained, it is impossible to perceive what the letter is any more, showing the importance of phase in perception of shapes. By looking at the original images side by the side with the 0 phase ones, the overall shading of the image is still maintained, just not properly corresponding to the original shape.

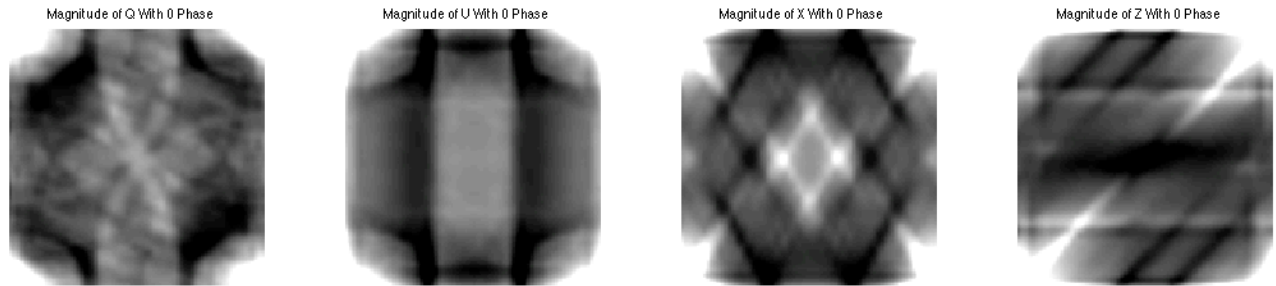


Fig. 3. Magnitude Maintained, 0 Phase

C. Keep Magnitude, Swap Phase With Another Letter

When the phase is swapped with another letter, the perceived image assumes the shape of the swapped image, again emphasizing importance of phase for perception of shape. The shading of the original image is once again maintained.

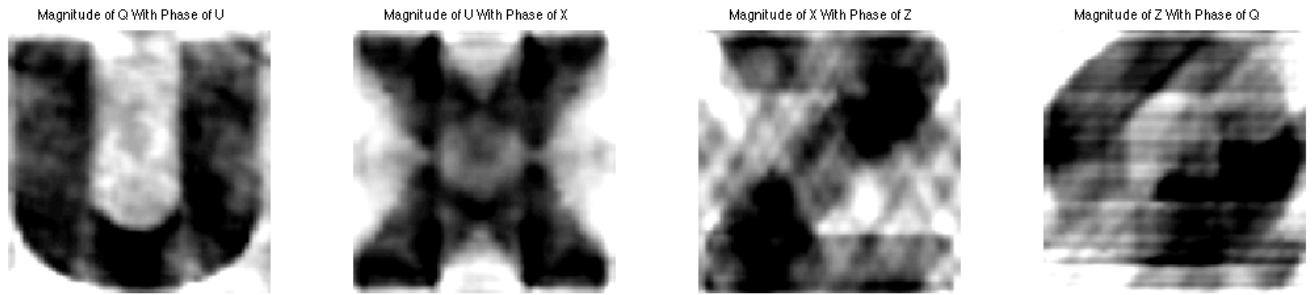


Fig. 4. Magnitude Swapped With Phase of Another Image

D. Keep Phase, Swap Magnitude With Another Letter

Again, when the phase is maintained, the original letter can be perceived, but the shading of the swapped magnitude appears.

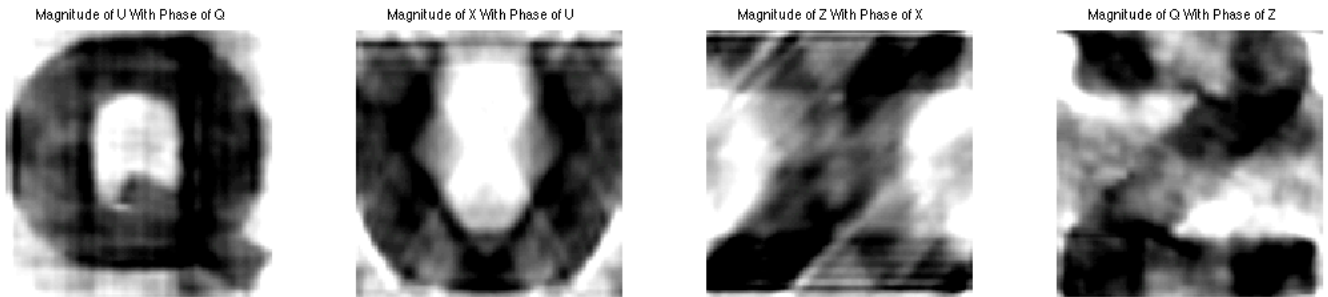


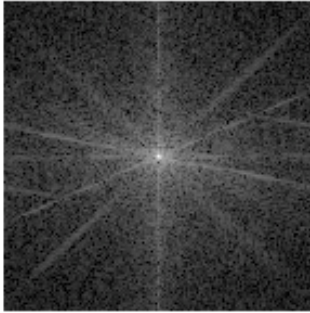
Fig. 5. Magnitude Swapped And Phase Maintained

III. IDEAL FILTERING IN FREQUENCY DOMAIN

A. Ideal Low Pass Filter With Cut Off Frequency Set Based on Power Maintained

A common method to set frequency cutoffs for image filters is based on percent of overall power in the pass region. This requires the percent of the power spectrum at all points in the DFT to be calculated. Ideal filters were then constructed by making matrices of unity gain if the power spectrum at the current point is less than the desired cutoff power spectrum, and 0 outside of the power spectrum.

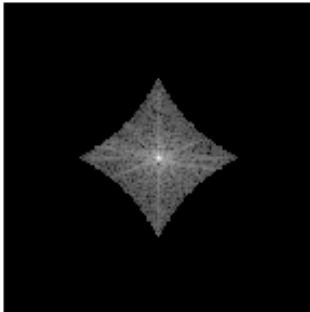
Unfiltered Magnitude w/ powThresh of 0.92



Unfiltered Image w/ powThresh of 0.92



Filtered Magnitude w/ powThresh of 0.92

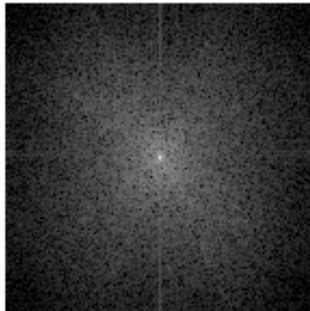


Filtered Image w/ powThresh of 0.92



Fig. 6. Camera man photo using Ideal Low Pass Filter preserving 92% of overall power.

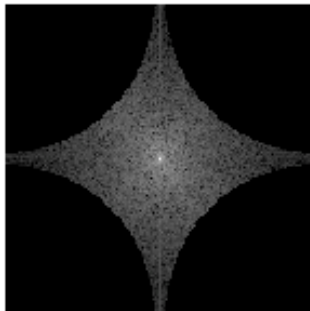
Unfiltered Magnitude w/ powThresh of 0.92



Unfiltered Image w/ powThresh of 0.92



Filtered Magnitude w/ powThresh of 0.92

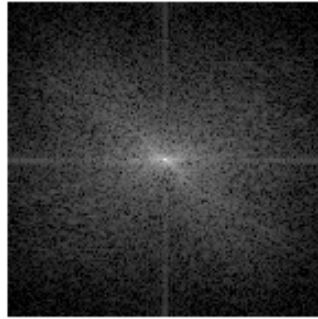


Filtered Image w/ powThresh of 0.92



Fig. 7. Arial view photo using Ideal Low Pass Filter preserving 92% of overall power.

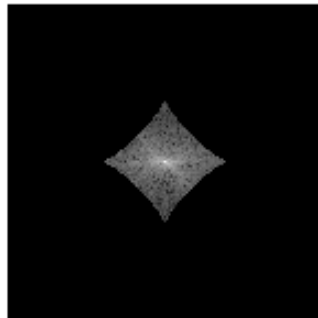
Unfiltered Magnitude w/ powThresh of 0.92



Unfiltered Image w/ powThresh of 0.92



Filtered Magnitude w/ powThresh of 0.92

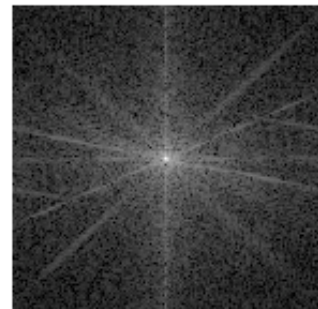


Filtered Image w/ powThresh of 0.92



Fig. 8. Lena photo using Ideal Low Pass Filter preserving 92% of overall power.

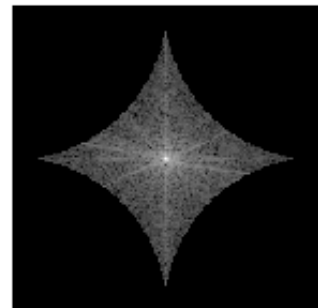
Unfiltered Magnitude w/ powThresh of 0.95



Unfiltered Image w/ powThresh of 0.95



Filtered Magnitude w/ powThresh of 0.95

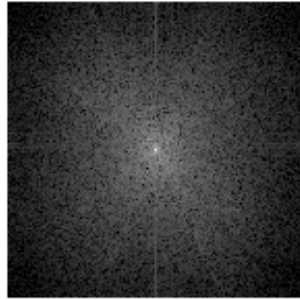


Filtered Image w/ powThresh of 0.95



Fig. 9. Camera man photo using Ideal Low Pass Filter preserving 95% of overall power.

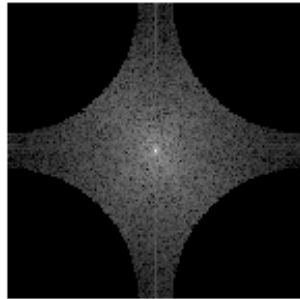
Unfiltered Magnitude w/ powThresh of 0.95



Unfiltered Image w/ powThresh of 0.95



Filtered Magnitude w/ powThresh of 0.95

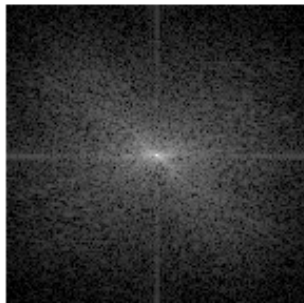


Filtered Image w/ powThresh of 0.95



Fig. 10. Aerial view photo using Ideal Low Pass Filter preserving 95% of overall power.

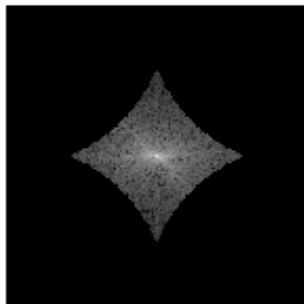
Unfiltered Magnitude w/ powThresh of 0.95



Unfiltered Image w/ powThresh of 0.95



Filtered Magnitude w/ powThresh of 0.95

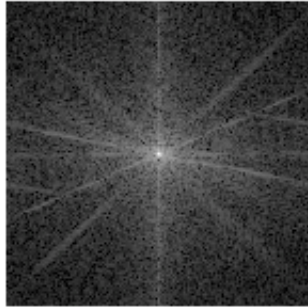


Filtered Image w/ powThresh of 0.95



Fig. 11. Lena photo using Ideal Low Pass Filter preserving 95% of overall power.

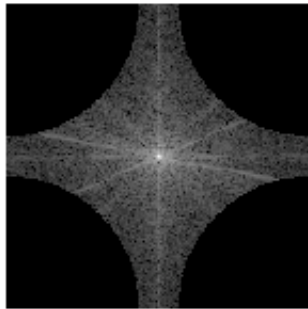
Unfiltered Magnitude w/ powThresh of 0.98



Unfiltered Image w/ powThresh of 0.98



Filtered Magnitude w/ powThresh of 0.98

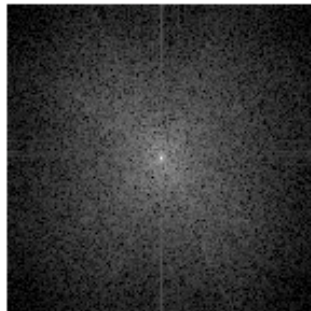


Filtered Image w/ powThresh of 0.98



Fig. 12. Camera man photo using Ideal Low Pass Filter preserving 98% of overall power.

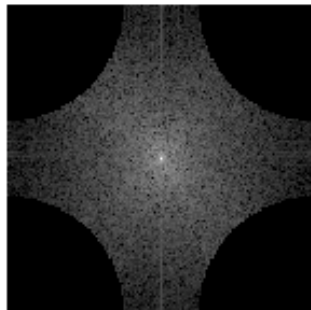
Unfiltered Magnitude w/ powThresh of 0.98



Unfiltered Image w/ powThresh of 0.98



Filtered Magnitude w/ powThresh of 0.98

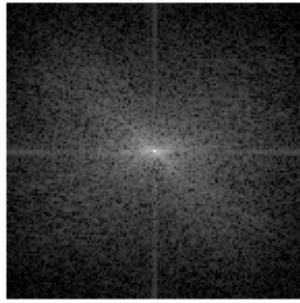


Filtered Image w/ powThresh of 0.98



Fig. 13. Aerial view photo using Ideal Low Pass Filter preserving 98% of power.

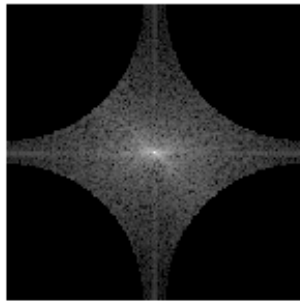
Unfiltered Magnitude w/ powThresh of 0.98



Unfiltered Image w/ powThresh of 0.98



Filtered Magnitude w/ powThresh of 0.98



Filtered Image w/ powThresh of 0.98



Fig. 14. Lena photo using Low Pass Filter preserving 98% of power.

Although the ideal filter has desirable effects in the frequency domain, it has some visual artifacts in the spatial domain. This is because the rectangle function and sinc function are Fourier Transform pairs, and so use of the ideal filter in the frequency domain creates a visual “ringing” effect in the spatial domain. These can be easily seen in Fig. 6 and Fig 8 where the power is 92%, as well as Fig. 9 and Fig. 11 where power is 95%. The ringing effects are most easily visible in areas without many high frequencies. In Fig. 7 and 10. (the arial city view) the ringing is not visible because there is too much high frequency content for it to be noticable. There is no perceivable difference in the pictures with 98% of the power spectrum (Fig. 12-14), due to the energy of the content being filtered not being easily visible.

B. Ideal Band Pass Filter

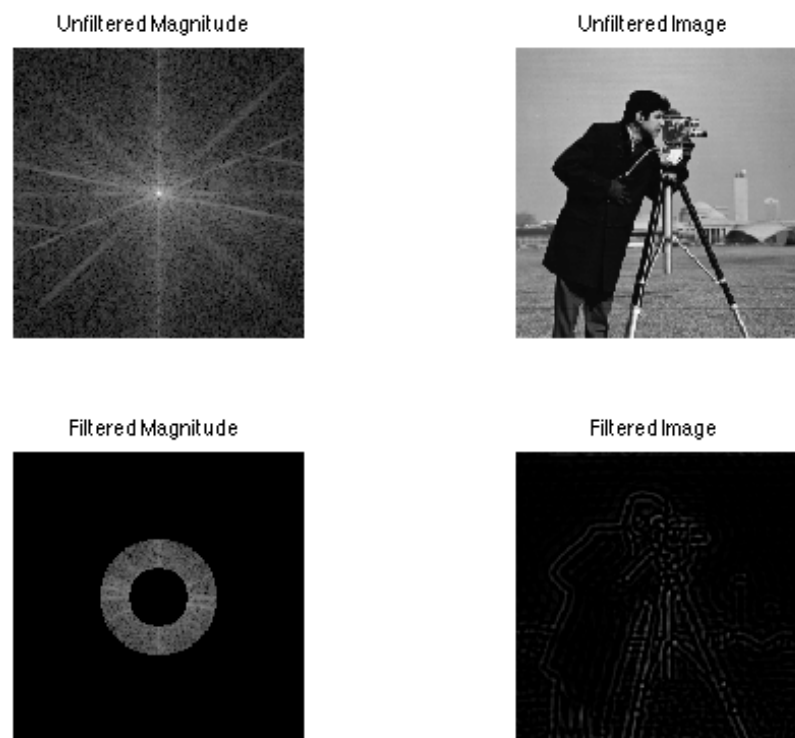


Fig. 15 Camera man photo using ideal band pass filter of frequency range .2 to .4.

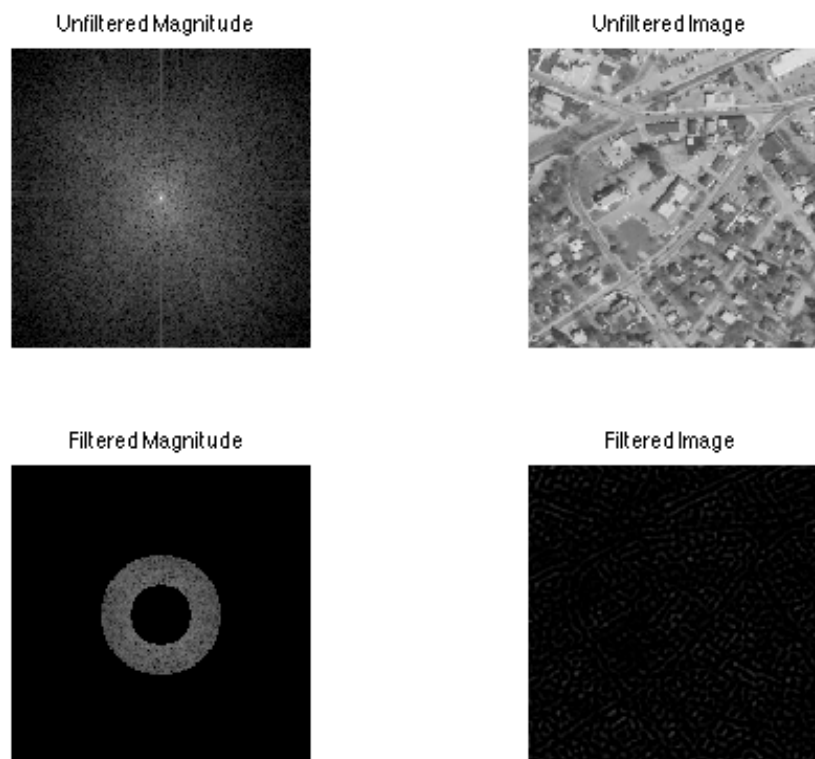


Fig. 16 Aerial view photo using ideal band pass filter of frequency range .2 to .4.

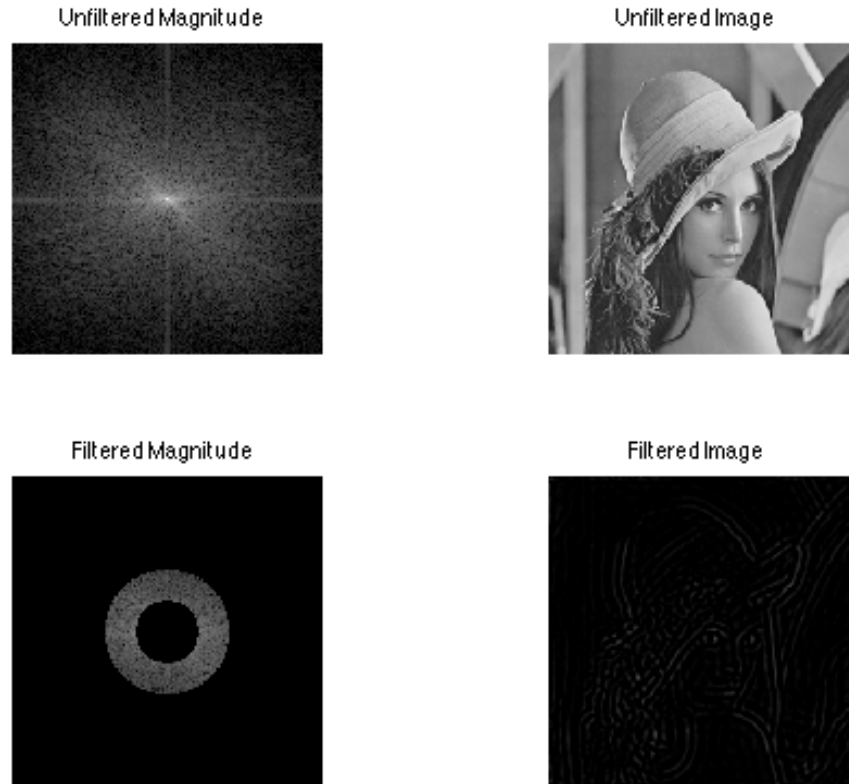


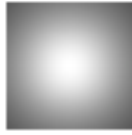
Fig. 17 Lena photo using ideal band pass filter of frequency range .2 to .4.

The most noticeable effect of the ideal band pass filter is that the images are very dark. This is due to the DC bin, which boosts the overall average intensity of the image being filtered. The leftover image is a faint outline of the shapes of the image. However, these outlines are slightly blurry, not completely clear. This is due to the high frequencies also being filtered out of the image. Leaving the DC bin in the image would have boosted the overall intensity of the image, but the contrast still would have been low due to lack of other frequency content. Although the band is the same range for all three images, the amount of power in this band is very different for each image. For the arial view, the band contains 75.22% of the power spectrum. The camera man photo contains 30.44% and the Lena photo contains 25.39% of the spectrum. The large amount in the camera man photo can be attributed to the amount of change, which is represented by high frequencies. The camera man and Lena picture are made up mostly of solid objects without much texture, and so there is not much high frequency energy.

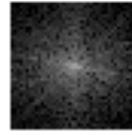
IV. BUTTERWORTH FILTERING

A. Various Orders of Low Pass Filtering to Maintain Power Spectrum

1 Ord Butterworth LPF



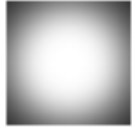
1 Ord Butterworth LPF Mag Response



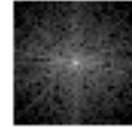
1 Filtered Image



2 Ord Butterworth LPF



2 Ord Butterworth LPF Mag Response



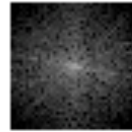
2 Filtered Image



3 Ord Butterworth LPF



3 Ord Butterworth LPF Mag Response



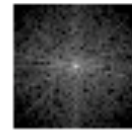
3 Filtered Image



4 Ord Butterworth LPF



4 Ord Butterworth LPF Mag Response



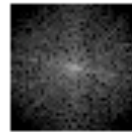
4 Filtered Image



5 Ord Butterworth LPF



5 Ord Butterworth LPF Mag Response

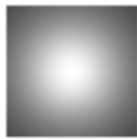


5 Filtered Image

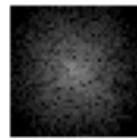


Fig. 18 Camera man photo using $N = 1:5$ order Butterworth Low Pass Filters with 95% power spectrum.

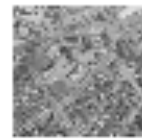
1 Ord Butterworth LPF



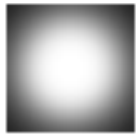
1 Ord Butterworth LPF Mag Response



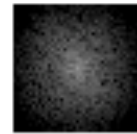
1 Filtered Image



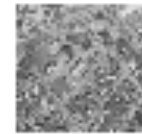
2 Ord Butterworth LPF



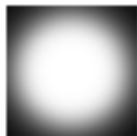
2 Ord Butterworth LPF Mag Response



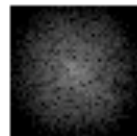
2 Filtered Image



3 Ord Butterworth LPF



3 Ord Butterworth LPF Mag Response



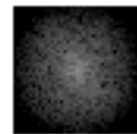
3 Filtered Image



4 Ord Butterworth LPF



4 Ord Butterworth LPF Mag Response



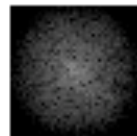
4 Filtered Image



5 Ord Butterworth LPF



5 Ord Butterworth LPF Mag Response



5 Filtered Image



Fig. 19 Camera man photo using $N = 1:5$ order Butterworth Low Pass Filters with 95% power spectrum.

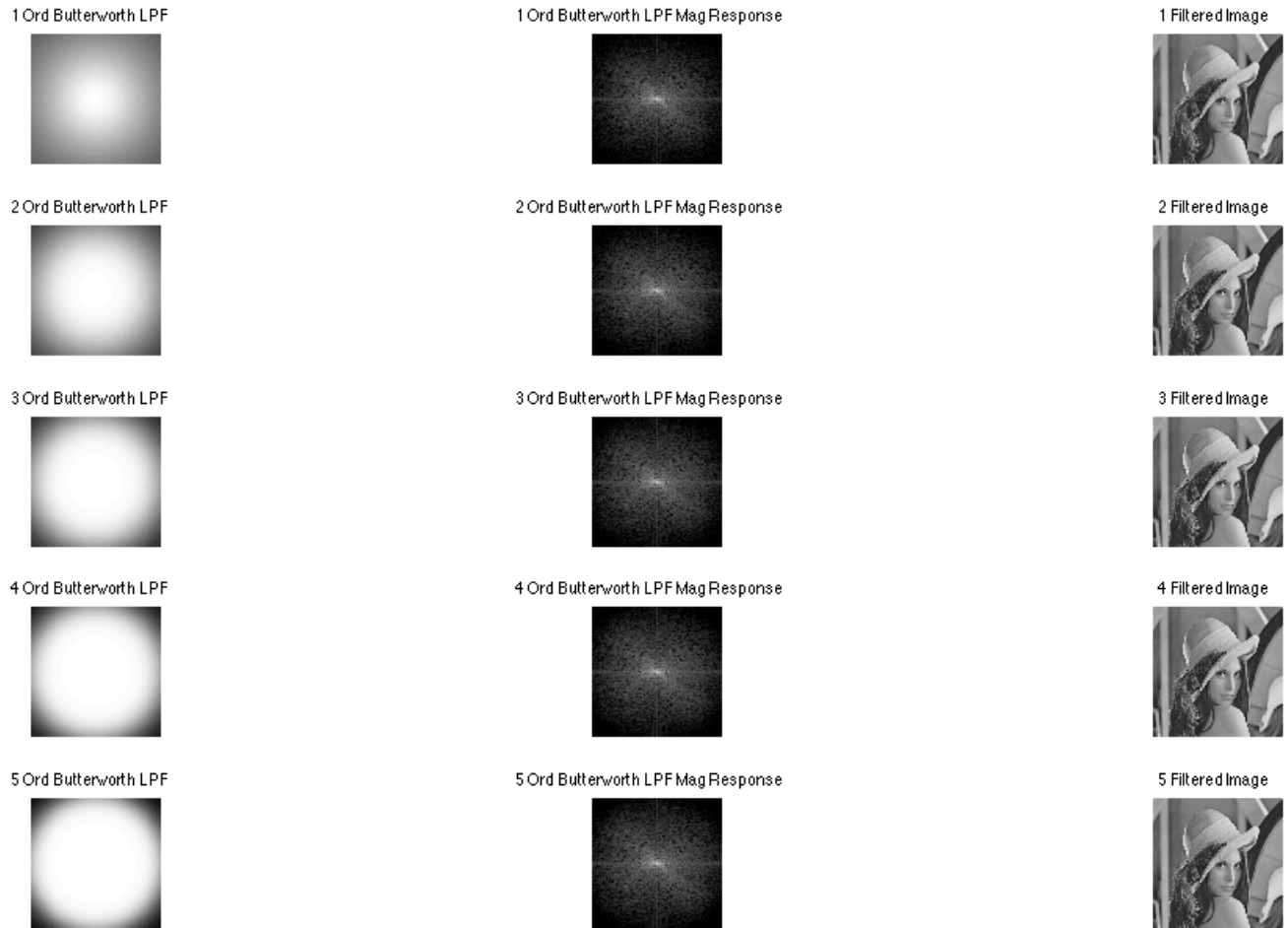


Fig. 20 Lena photo using $N = 1:5$ order Butterworth Low Pass Filters with 95% power spectrum.

For all of the images using Butterworth Low Pass Filters with 95% power spectrum, there is no noticeable blurring or distortion to the image. The lack of ringing can be attributed to the smoothness of the Butterworth Filter in the frequency domain, creating a smooth response without ringing in the spatial domain. The lack of visible blurring shows that all of the visible changes in Fig. 9 and 10 are due to ringing and not any visible blurring.

B. Butterworth High Pass Filtering

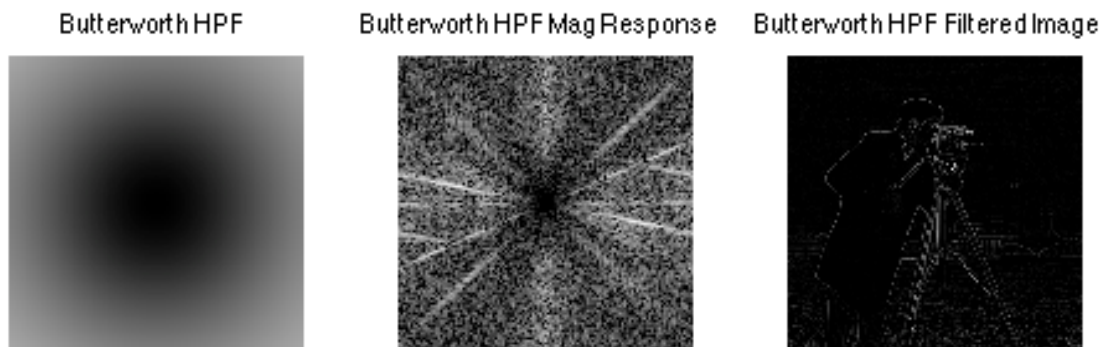


Fig. 21 Camera man photo using Butterworth High Pass Filter with upper 5% of power spectrum.

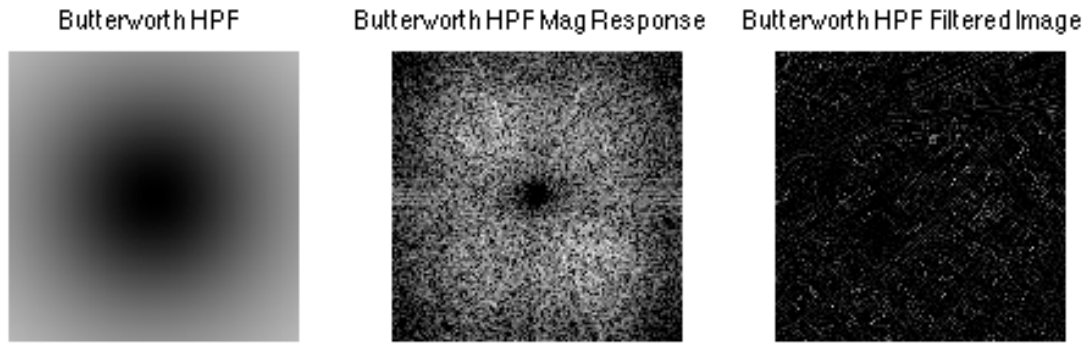


Fig. 22 Aerial view photo using Butterworth High Pass Filter with upper 5% of power spectrum.

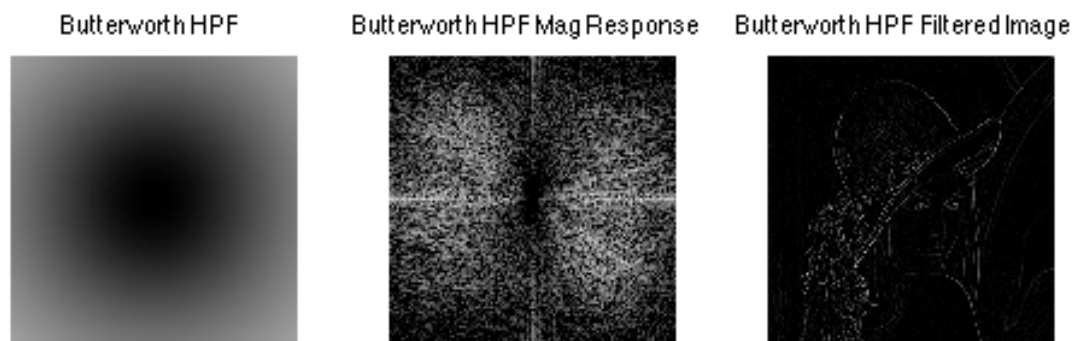


Fig. 23 Lena photo using Butterworth High Pass Filter with upper 5% of power spectrum.

The High Pass Filtered Images have similar characteristics to the band passed images in that they are very dark due to lack of DC content. However, the outline of the figures in the images is much clearer due to the presence of high frequency content, and also because a Butterworth Filter is being used so there is no ringing.

C. Adjusted Low Pass Butterworth

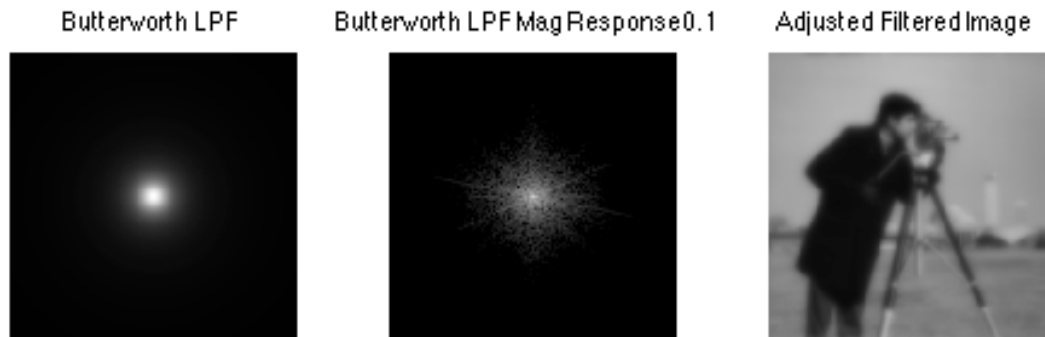


Fig. 24 Camera man photo using Butterworth Low Pass Filter of .1.

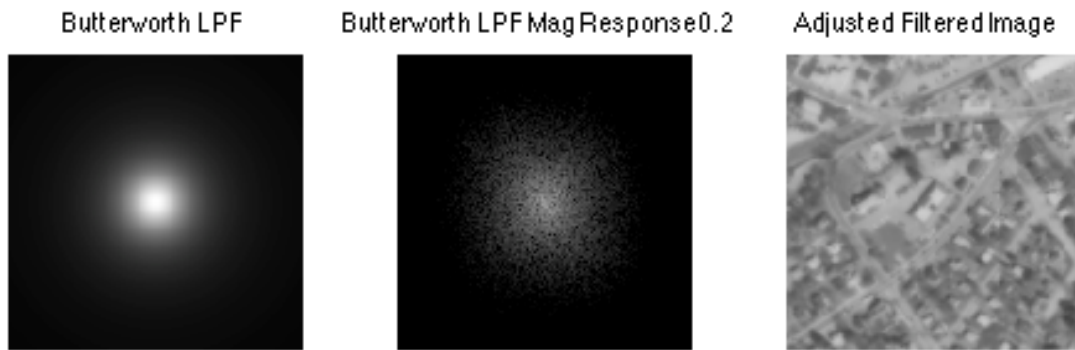


Fig. 25 Camera man photo using Butterworth Low Pass Filter of .2.

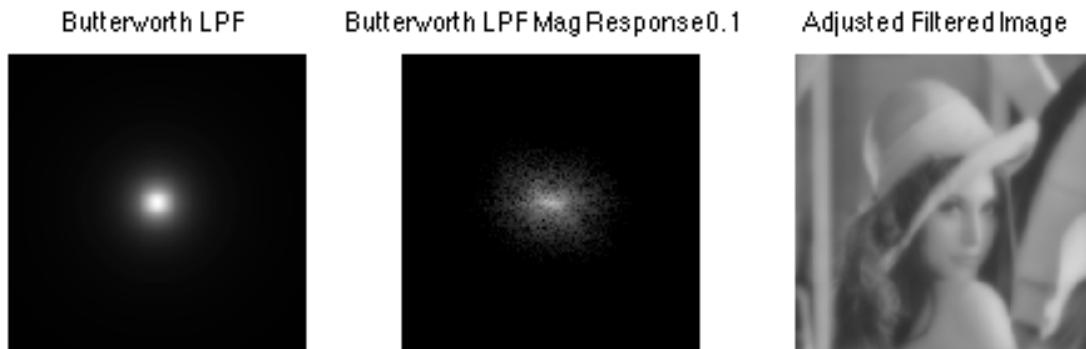


Fig. 25 Camera man photo using Butterworth Low Pass Filter of .1.

In Fig. 23-25 low pass cutoff frequencies were chosen that create a visible blur to the image, but still allow the import details of the image to be perceived. As noted earlier, the Arial View photo has a much higher percentage of frequency content in the higher frequency bands then do the Lena photo and Camera Man photo. This data is supported by the frequencies that were set in Fig. 23-25. In Fig. 23 and 25, the low pass cutoff was set to .1, but for Fig. 24, it was set to .2. This is again due to the high frequency content and detail in the Arial View photo, while most of the other two images are not as texturally complex and have less change.

D. High-Boost Butterworth Filter

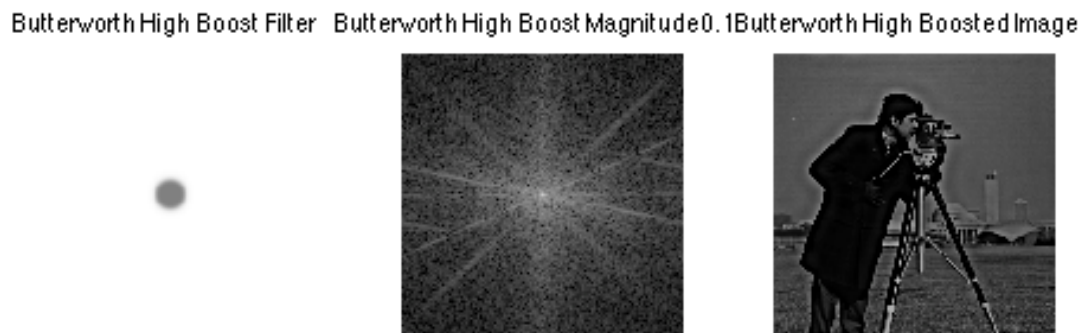


Fig. 26 Camera man photo using High-Boost Butterworth Filter

Butterworth High Boost Filter Butterworth High Boost Magnitude 0.2 Butterworth High Boosted Image

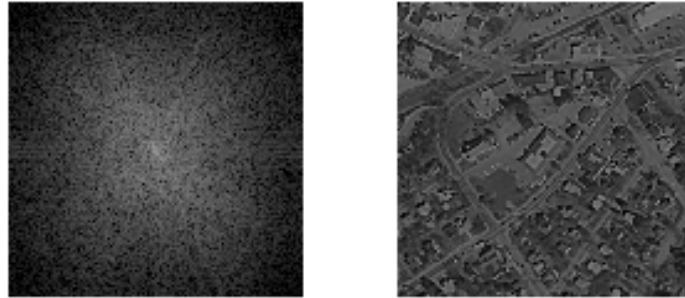


Fig. 27 Aerial view photo using High-Boost Butterworth Filter

Butterworth High Boost Filter Butterworth High Boost Magnitude 0.1 Butterworth High Boosted Image



Fig. 28 Lena photo using High-Boost Butterworth Filter

The High-Boost Filter is created by adding a high pass filtered version of the image to the magnitude of the original image and scaling it accordingly. Since a high pass filter can be created by $1 - \text{LPF}$, adding another factor of one to represent the original magnitude will give $2 - \text{LPF}$. By making the factor '2' a variable, the ratio of the original magnitude and the high pass filter can be controlled. For this example though, the factor was kept at 2, so that the original magnitude and high frequency content were at equal levels. While the images are much visibly sharper than the original, the tradeoff is that they are darker. This is because the high passed values are added directly over the previous values and the image has to be rescaled, lowering the DC intensity level.

IV. CONCLUSION

The main takeaways from this project were the importance of phase in recognizing shapes, and the various tradeoffs that must be made when processing images. While ideal filters allow precise removal of frequencies without attenuation, there is visible ringing present in the spatial domain. While Butterworth filters do not have this ringing, they are not as accurate at filtering specific frequencies. While low pass filtering can be used to remove noise and retains most of the power spectrum, it also blurs the image and makes it harder to properly recognize content. High pass filters can be used to sharpen the edges of images, but these will lower the power spectrum and lower the overall brightness of the image. Proper image processing is done by balancing the need to fix problems with images, without introducing new artifacts to them.