Interpolated FIR Filter Techniques

Connor A. McCullough

Music Engineering/Electrical Engineering

University of Miami, Coral Gables FL, 33146

Abstract—**While Finite Impulse Response filters are valuable for their properties of guaranteed stability and option for linear phase, their main drawback is the computational resources necessary to implement such high order filters, especially those with steep transition bands. In the mid-80's, filter design methods were proposed to decrease order of filters by interpolating the model filter. Later developments included optimization functions for the "stretch" length of the filter, implementing multiple IFIR filters within each other to maximize filter size reduction, and decreasing the number of necessary coefficient multipliers.**

I. Introduction

The two types of digital filters, Finite Impulse Response (FIR) and Infinite Impulse Response (IIR) both have various benefits and drawbacks. Finite Impulse Response filters are always stable, can have linear phase, but have large computation requirements, especially when the transition band is narrow. Although IIR filters are much more efficient in terms of required computation, they are not always stable and cannot have linear phase, making them impractical for certain purposes. Due to redundancy in FIR coefficients, an FIR can be implemented in two cascaded sections: a sparse set of impulse response values, and an interpolation function, which will greatly reduce the number of multipliers and adders as a ratio of the amount of zero values added between impulse response values. This technique was first proposed in the 1980's, and since then there has been much more progress with this method. Advancements since this time have included use of decimation in IFIR design, optimization of the filter design, stretching factors, multiplier free FIR design.

II. Early Interpolated Finite Impulse Response Methods

Design of IFIR filters starts with a digital model filter $H_M(Z)$. In the time domain, L-1 zero valued samples are inserted between the current original samples to give $h'_M(n)$. The Z-transform, $H'_M(Z)$ is equivalent to $H_M(Z^L)$ [1].

In order to obtain the interpolated IFIR transfer function $H_i(Z)$, $H'_M(Z)$ must be cascaded with an interpolator, G(Z). The adding of L-1 zero values between the original impulse response values compresses the frequency response by 1/L and replicates it with a period of 2π/L. G(Z) must therefore be designed so that all the periodic replicas are attenuated below the stopband ripple value.

A variety of values can be chosen for L, but there is a maximum value for L given by:

$$L_{MAX} = \left\lfloor \frac{\pi}{\omega_{SL}} \right\rfloor \text{ (LPF)} \tag{1}$$

which ensures the model filters stopband edge is less than pi. This maximum value has its equivalents for high pass and bandpass filters:

$$L_{MAX} = \left\lfloor \frac{\pi}{\pi - \omega_{SH}} \right\rfloor \text{ (HPF)} \tag{2}$$

$$L_{MAX} = \left\lfloor \frac{2\pi}{\omega_{S1} - \omega_{S2}} \right\rfloor \text{ (BPF)} \tag{3}$$

III. Optimal IFIR Filters

One of the main shortcomings of early IFIR design methods was lack of a way of analytically optimizing the stretching factor L. Mehrnia and Wilson observed that as L increases, the order of the model filter decreased, but the order of the image suppressor (referred to as interpolated G(Z) in earlier publications) increased[2]. By starting with the Kaiser formula, they obtain a function for optimal value of L:

$$L_{OPT} = \frac{2\pi}{\omega_p + \omega_s + \sqrt{2\pi(\omega_s - \omega_p)}} \tag{4}$$

Mehrnia and Wilson also propose a method in which the image suppressor filter is itself an IFIR filter with a secondary image suppressor filter with the overall transfer function:

$$H_i(Z) = H_M(Z^L)[G_1(Z^L)G_2(Z^L)] \tag{5}$$

The function for total multipliers became a function of both L and L1 with total number of multipliers given by:

$$f(L,L_1)=$$

$$L_1 \le L \le \frac{2\pi}{(\omega_S+\omega_P)}$$

$$\frac{1}{L(\omega_S-\omega_P)}+\frac{1}{L_1[(\frac{2\pi}{L}-\omega_S)-\omega_P]}+\frac{1}{\frac{2\pi}{L_1}-(\frac{2\pi}{L}-\omega_S)-\omega_P} \quad (6)$$

The above method of modeling the image suppressor filter as another IFIR can be done multiple times with three, four, or even more values of L to optimize. However, there must also be a method for determining if implementing an IFIR filter will provide significant reductions in multipliers, which Mehrnia and Wilson also give:

$$L_1 \le L \le \frac{2\pi}{(\omega_S+\omega_P)} \quad (7)$$

IV. Multiplier Free IFIR

Another method for reducing the number of computations of an IFIR filter is to reduce the total number of multipliers needed to represent the coefficients (proposed by Dolocek and Mitra)[3]. The goal of this design process is that the filter still meets the original design criteria, but all coefficients can be represented with minimum number of signed powers of two. The first step of this process is design of an IFIR filter as possible, followed by application of a rounding function:

$$h_r(n)=r\bullet h_I(n)=r\bullet round(h(n)/r) \quad (8)$$

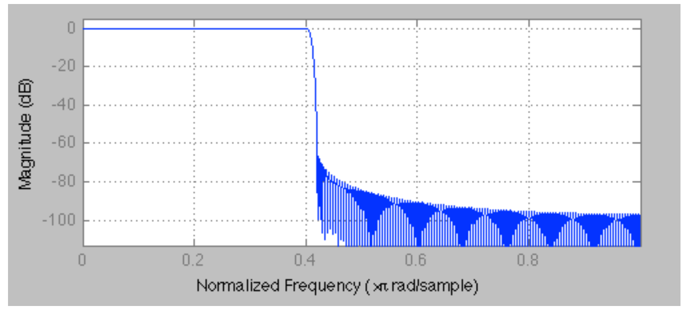
The rounding in this function is often set to r = 0.01 as a starting point. Due to distortion in both the passband and stopband responses following rounding, additional sharpening polynomial functions may be necessary, starting with parameters m = 1 and n = 1. If the filter does not meet design criteria, r can be increased, and then if neccesary m and n can be increased. While Dolocek and Mitra do address a key issue in computational costliness of IFIR filters, they do not provide functions to optimize the parameters of their method and it is left to the designer to go back and redesign the filter if the applied parameters do not satisfy the filter criteria.

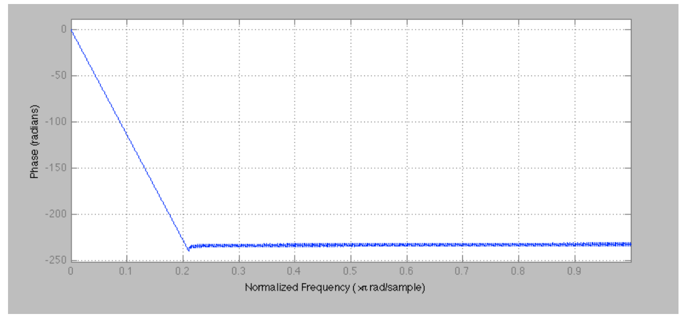
V. Example Using Optimal FIR Design

This example will demonstrate that IFIR filters have significantly decreased order compared to FIRs designed without interpolation. Parameters:

$$H(Z):\omega_P=0.20\pi,\delta_p=0.001$$

$$\omega_S=0.21\pi,\delta_S=0.001 \quad (9)$$





The filter above was designed using the Parks-McClellan algorithm. The order was 726. Optimal interpolation factor:

$$L_{OPT}=\left[\frac{2\pi}{0.21\pi+0.20\pi+\sqrt{2\pi(0.21\pi-0.20\pi)}}\right]=2 \quad (10)$$

Calculations for model filter component of IFIR:

$$H_M(Z^L):\omega_p^L=L\omega_p=0.40\pi,\delta_P^L=\frac{\delta_p}{2}=0.0005$$

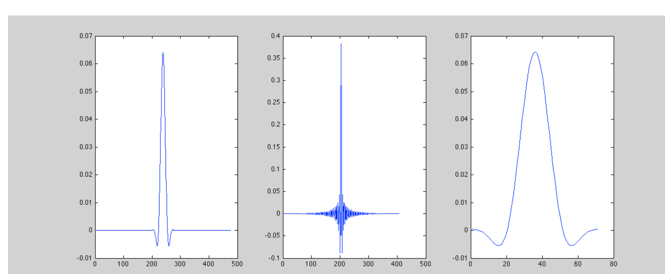
$$\omega_s^L=L\omega_S=0.42\pi,\delta_S^L=\delta_S=0.001 \quad (11)$$

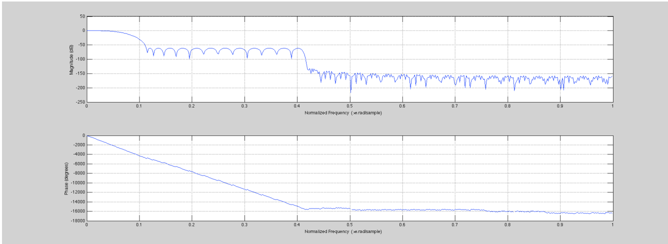
Calculations for image filter component:

$$G(Z):\omega'_p=\omega_p=0.20\pi,\delta'_p=\frac{\delta_p}{2}=0.0005$$

$$\omega'_S=\frac{2\pi}{L}-\omega_S=.79\pi,\delta'_s=\delta_s=0.001 \quad (12)$$

Order (model component) = 404
Order (image component) = 70
Total = 474
Reduction = 38%





REFERENCES

[1] Y. Neuvo, D. Cheng-Yu, and S.K. Mitra, "Interpolated Finite Impulse Response Filters" *IEEE Trans. Acoustics, Speech, and Signal Processing,* vol. ASSP-32, pp.563-570, June 1984.

[2] A. Mehrnia and A. N. Willson, Jr., "On Optimal IFIR Filter Design"*ISCAS,* pp.133-136, 2004.

[3] G. Jovanovic-Dolecek and S.K. Mitra, "Multiplier-free FIR Filter Design Based on IFIR Structure and Rounding," IEEE, pp. 559-662, 2005.